Simulating Non-Hermitian Physics

易为

中国科学技术大学

安徽理工大学

Outline

- Emergence of non-Hermiticity
- Quantum simulation of non-Hermitian physics
  - Topological transfer
  - Non-Hermitian skin effect
- Summary and outlook

Non-Hermitian quantum systems?

- Hermiticity central to conventional quantum mechanics

$$\hat{A} = \hat{A}^\dagger \quad \Rightarrow \quad \text{Real eigenvalues}$$

- Parity-Time symmetric non-Hermitian models (Benders et al., 1998)

$$\begin{cases} \mathcal{PT} H \left(\mathcal{PT}\right)^{-1} = H \\ \mathcal{PT}|\psi\rangle \sim |\psi\rangle \end{cases} \Rightarrow E = E^*$$

- Non-Hermitian quantum mechanics in different contexts
  - Feshbach projection
  - Quantum trajectory description of open systems
  - Double-space description of open systems
  - Alien quantum mechanics
  - Quantum mechanics plus post selection

Feshbach projection

- A subsystem P coupled to an environment Q (often continuum)
- Project the dynamics into the subsystem

$$H_{\text{eff}}(E) = H_{PP} + H_{PQ}\frac{1}{E - H_{QQ}}H_{QP}$$
$$= H_{PP} + \Delta(E) - \frac{i}{2}\Gamma$$

- Non-Hermiticity from the poles of $\frac{1}{E-H_{QQ}}$
- Resonance between states in P and scattering states in Q
- Relevant for heavy nuclei, quantum resonance, and subradiance/superradiance

H. Feshbach, Ann. Phys. 5, 357 (1958)
Y. Ashida, Z. Gong, M. Ueda, Adv. Phys. 69, 3 (2020)

Open systems with Markovian reservoir

- Lindblad equation

$$\frac{d}{dt}\rho = -i\left[H,\rho\right] - \frac{1}{2}\Gamma\left(S^\dagger S\rho + \rho S^\dagger S - 2S\rho S^\dagger\right)$$
$$= -i\left(H_{\text{eff}}\rho - \rho H_{\text{eff}}^\dagger\right) + \Gamma S\rho S^\dagger$$
$$H_{\text{eff}} = H - \frac{i}{2}\Gamma S^\dagger S$$

- Quantum Langevin equation

$$\frac{d}{dt}A = i\left[H_{\text{eff}}, A\right] - i\langle F(t)\rangle$$

- Trace-conserving with quantum jump process
- Basic commutation relations kept intact by the Langevin noise

Quantum trajectory approach

- Evolve the state (stochastically), rather than the density matrix
- At each time step, roll a dice:
  - With probability $1-\delta p$

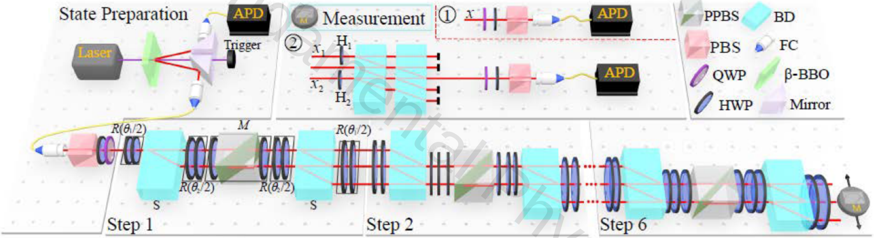
$$|\psi(t+\delta t)\rangle = \frac{1}{\sqrt{1-\delta p}}(1 - iH_{\text{eff}}\delta t)|\psi(t)\rangle$$

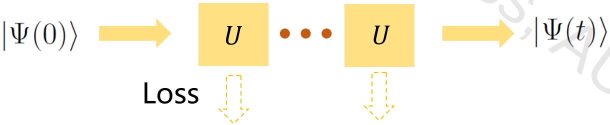
  - With probability δp (quantum jump)

$$|\psi(t+\delta t) = \frac{1}{\sqrt{\delta p/\delta t}}\sqrt{\Gamma}S|\psi(t)\rangle$$

  Here $\delta p = \Gamma\delta t\langle\psi(t)|S^\dagger S|\psi(t)\rangle$
- Dynamics driven by H_{eff} in the absence of quantum jumps
- Unconditional vs. conditional dynamics

Non-Hermiticity in single-photon interferometry (quantum walk)





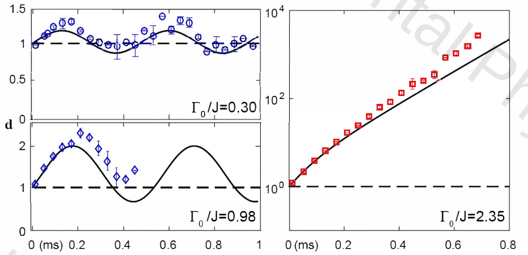
$$|\Psi(t)\rangle = U^t|\Psi(0)\rangle, \quad U = e^{-iH_{\text{eff}}}$$

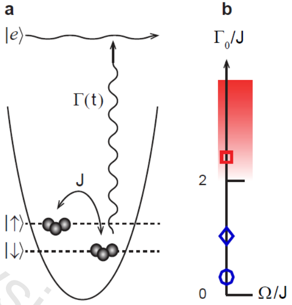
K. Wang et al., Nat. Commun. 10, 2293 (2019)

Non-Hermiticity in cold atoms: PT symmetry with cold atoms

- Passive PT symmetry

$$H(t) = J\sigma_x - i\Gamma|\downarrow\rangle\langle\downarrow| = i\frac{\Gamma}{2} + H_{\mathcal{PT}}$$





- Focusing on the remaining atoms (non-interacting)

J. Li, A. K. Harter, J. Liu, L. de Melo, Y. N. Joglekar, L. Luo, Nat. Commun. 10, 855 (2019)

Double-space formalism

- Mapping the density matrix and Lindblad equation

$$\rho = \sum_{mn} \rho_{mn} |m\rangle\langle n| \Rightarrow |\Psi\rangle = \sum_{mn} \rho_{mn} |m\rangle \otimes |n\rangle$$

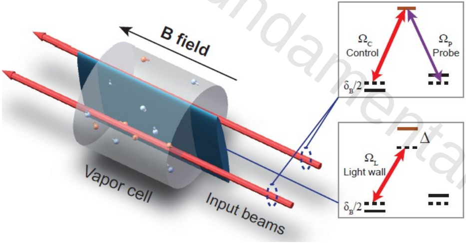
Lindblad equation $\Rightarrow (H_R - iH_I)|\Psi\rangle = E|\Psi\rangle$

$$H_R = H \otimes I - I \otimes H^T$$

$$H_I = -2S \otimes S^* + (S^\dagger S) \otimes I + I \otimes (S^\dagger S)^*$$

- Effective non-Hermitian Hamiltonian $H_R - iH_I$ drives dynamics in the double space
- Louisville gap encoded in E (how steady-state can be approached)

Non-Hermiticity in spin-waves (atomic vapour cell)



Optical Bloch equations

$$\begin{cases} \dot{\rho}_{12}^{(1)} = -\gamma_{12}'\rho_{12}^{(1)} + \Gamma_c \rho_{12}^{(2)} - \dfrac{\Omega_c^{(1)*}\Omega_p^{(1)}}{\gamma_{23}}, \\ \dot{\rho}_{12}^{(2)} = -\gamma_{12}'\rho_{12}^{(2)} + \Gamma_c \rho_{12}^{(1)} - \dfrac{\Omega_c^{(2)*}\Omega_p^{(2)}}{\gamma_{23}}, \end{cases}$$

Effective Hamiltonian

$$H \propto (g a^\dagger b - g^* b^\dagger a) e^{i\theta_0}$$

- Vapor cell as a gate with $U = e^{-iH\tau}$
- H inferred from light transport

P. Peng et al., Nat. Phys. 12, 1139 (2016)

X. Meng et al., Photonics Research (2022)

Non-Hermitian quantum mechanics

- Orthonormal basis in conventional quantum mechanics

$$\langle\psi_n|\psi_m\rangle = \delta_{mn}, \quad \sum_n |\psi_n\rangle\langle\psi_n| = I$$

- Non-Hermitian version: left/right eigenstates

$$\langle\psi_n|\chi_m\rangle = \delta_{mn}, \quad \sum_n |\psi_n\rangle\langle\chi_n| = I$$

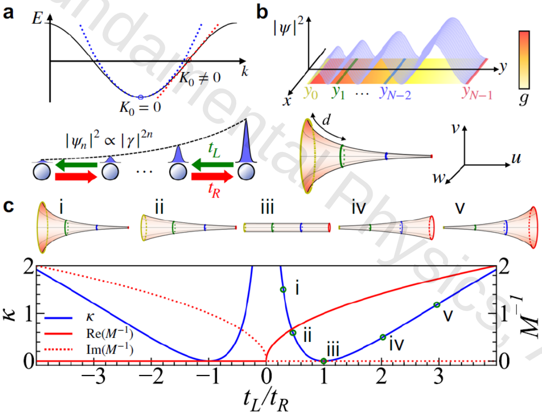
$$\text{with} \quad H_k|\psi_\mu\rangle = \epsilon_\mu|\psi_\mu\rangle, \quad H_k^\dagger|\chi_\mu\rangle = \epsilon_\mu^*|\chi_\mu\rangle$$

- Redefine dual space and inner product through the metric $\eta = \sum_n |\chi_n\rangle\langle\chi_n|$

$$|\chi_n\rangle = \eta|\psi_n\rangle \Rightarrow |\phi^{(i)}\rangle = \sum_n d_n^{(i)}|\psi_n\rangle, \quad \langle\phi^{(i)}_{\text{dual}}| = \sum_n d_n^{(i)*}\langle\chi_n|$$

$$\langle\phi^{(1)}_{\text{dual}}|\phi^{(2)}\rangle = \sum_n d_n^{(1)*}d_n^{(2)} = \langle\phi_r^{(1)}|\eta|\phi_r^{(2)}\rangle$$

Alien quantum mechanics: curved space



- Hatano-Nelson $\leftrightarrow$ a strip on a 2D curved surface

C. Lv, R. Zhang, Z. Zhai, Q. Zhou, Nat. Commun. 13, 2184 (2022)

Chiral state transfer near the exceptional point

- $\mathcal{PT}$ symmetry

  $H \neq H^\dagger \Rightarrow E \in \mathbb{C}$, however, under $\mathcal{PT}$ symmetry

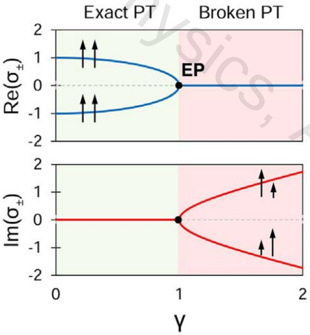
$$\begin{cases} \mathcal{PT} H (\mathcal{PT})^{-1} = H \\ \mathcal{PT}|\psi\rangle \sim |\psi\rangle \end{cases} \Rightarrow E = E^*$$

C. M. Bender and S. Boettcher, Phys. Rev. Lett. **80**, 5243 (1998)

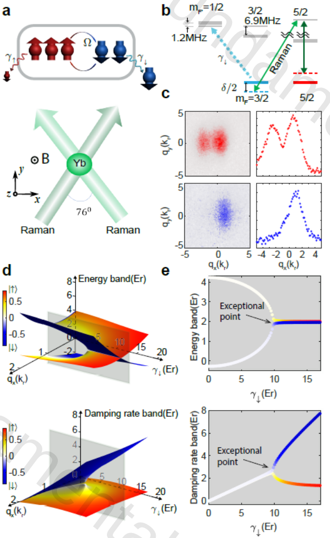
- $\mathcal{PT}$ transition and the exceptional point

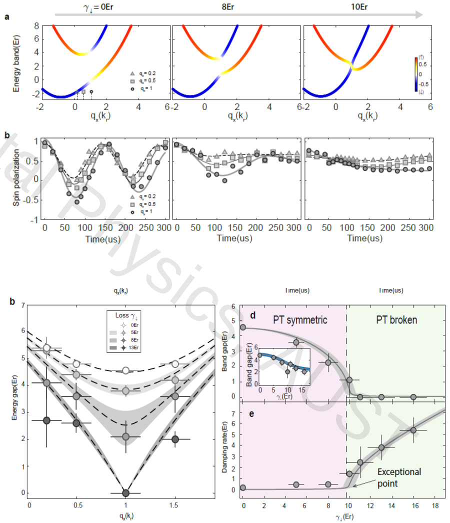
non-Hermitian matrix

$$\begin{pmatrix} \omega - i\gamma & g \\ g & \omega + i\gamma \end{pmatrix}$$



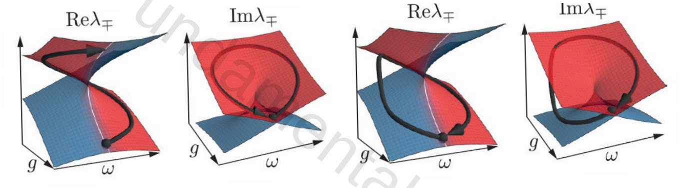
Exceptional points in a Fermi gas

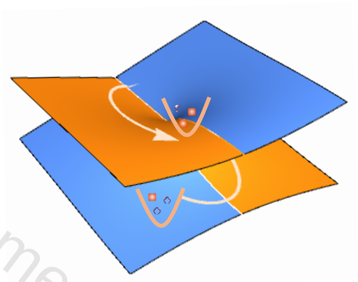




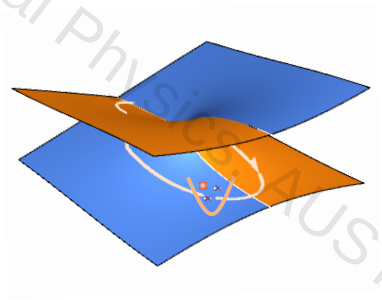
Z. Ren et al., Nat. Phys. 18, 385 (2022)

Collective chiral transfer



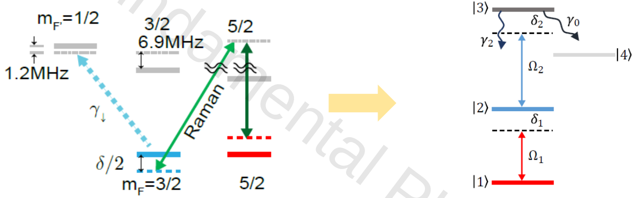


CCW: adiabatic, state flip



CW: non-adiabatic, non-flip

Decoherence as additional jump processes



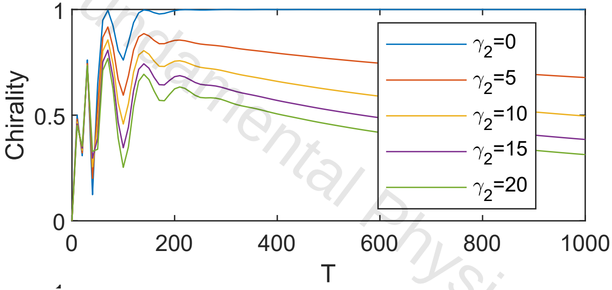
Lindblad equation

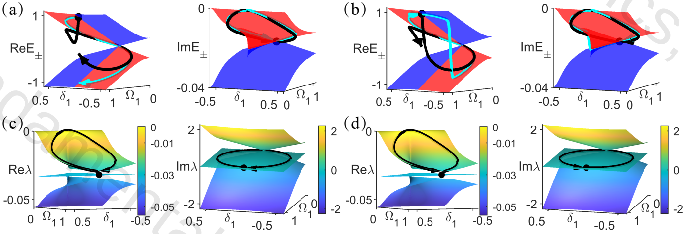
$$\dot{\rho} = -i(H_{\text{eff}}\rho - \rho H_{\text{eff}}^{\dagger}) + L_{\phi}\rho L_{\phi}^{\dagger} - \frac{1}{2}L_{\phi}^{\dagger}L_{\phi}\rho - \frac{1}{2}\rho L_{\phi}^{\dagger}L_{\phi},$$

with

$$H_{\text{eff}} = H_0 - i\Gamma|2\rangle\langle 2|, \quad L_{\phi} = \sqrt{\gamma_{\phi}}\,|2\rangle\,\langle 2|\,.$$

Consequences for the chiral transfer

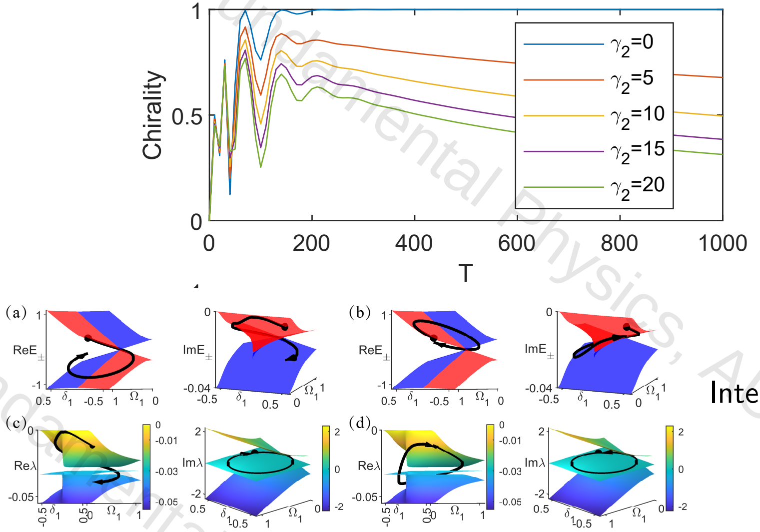




$T \to \infty$

K. Sun and WY, Phys. Rev. A 108, 013302 (2022)

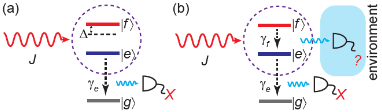
Consequences for the chiral transfer

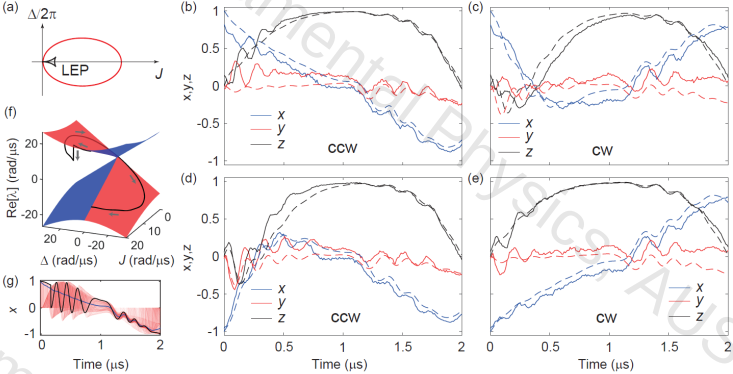


Intermediate T

K. Sun and WY, Phys. Rev. A 108, 013302 (2022)

With superconducting qubits

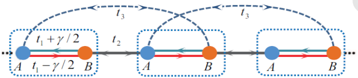


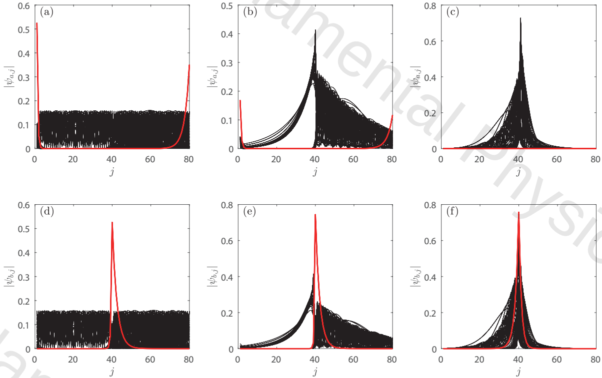


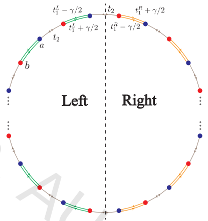
W. Chen et al., Phys. Rev. Lett. 127, 140504 (2021)

W. Chen et al., Phys. Rev. Lett. 128, 110402 (2022)

Non-Hermitian skin effects



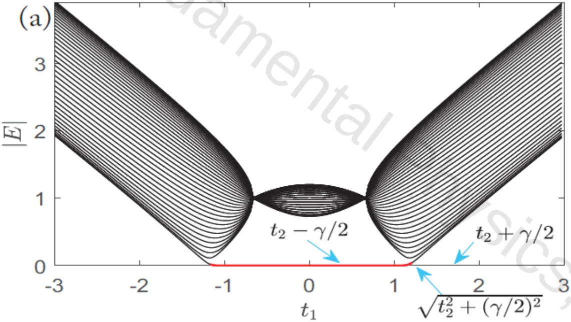




S. Yao and Z. Wang, Phys. Rev. Lett. 121, 086803 (2018)

T.-S. Deng and WY, Phys. Rev. B 100, 035102 (2019)

Failure of bulk-boundary correspondence



- Bulk topological invariants fail to predict edge states
- A solution: non-Bloch band theory

S. Yao and Z. Wang, Phys. Rev. Lett. 121, 086803 (2018)

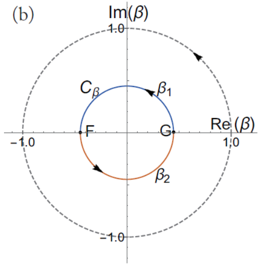
K. Yokomizo and S. Murakami, Phys. Rev. Lett. 123, 066404 (2019)

Non-Bloch band theory to the rescue

- Bulk-state ansatz

$$|\Psi\rangle = \sum_j \left(\psi_{a,j} a_j^\dagger + \psi_{b,j} b_j^\dagger\right)|0\rangle$$

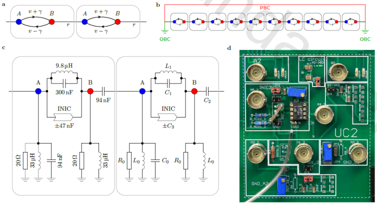
$$\begin{pmatrix} \psi_{a,j} \\ \psi_{b,j} \end{pmatrix} = \begin{pmatrix} \phi_a^{(1)} \\ \phi_b^{(1)} \end{pmatrix} \beta_1^j + \begin{pmatrix} \phi_a^{(2)} \\ \phi_b^{(2)} \end{pmatrix} \beta_2^j$$



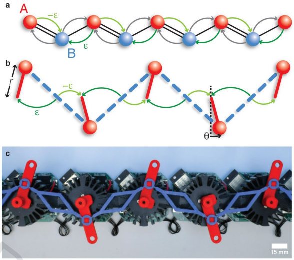
- Hermitian limit: $\beta_{1,2} = e^{\pm ik}$
  Non-Hermitian (open boundary): $|\beta_{1,2}| \neq 1$
- Non-Bloch topological invariants:
  Winding number calculated over the deformed Brillioun zone
- More generally, $\beta(k,t)$ in more complicated systems

S. Yao, F. Song, Z. Wang, Phys. Rev. Lett. 121, 136802 (2018)

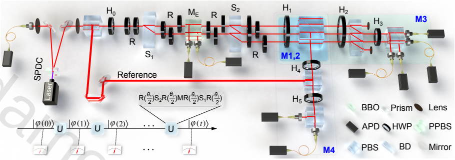
Experiments on skin effects/b.b.c.



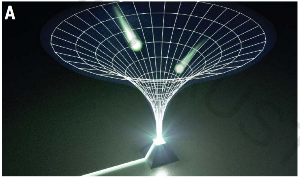
Topoelectrical circuits: Nat. Phys. 16, 747 (2020)



Mechanics: PNAS 117, 29561 (2020)

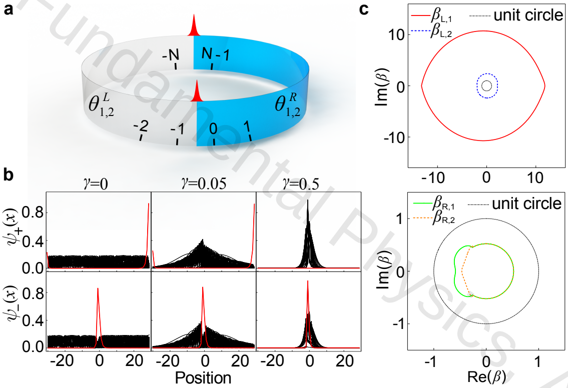


Photonic quantum walk: Nat. Phys. 16, 761 (2020)



Optical fibre: Science 368, 311 (2020)

Simulating non-Hermitian skin effect with QWs



- Floquet operator $U = R(\frac{\theta_1}{2})S_2R(\frac{\theta_2}{2})MR(\frac{\theta_2}{2})S_1R(\frac{\theta_1}{2})$ with

$$M = \mathbb{1}_w \otimes \left(e^{\gamma}|\uparrow\rangle\langle\uparrow| + e^{-\gamma}|\downarrow\rangle\langle\downarrow|\right)$$

L. Xiao, T. Deng, K. Wang, G. Zhu, Z. Wang, WY, P. Xue, Nat. Phys. 16, 761 (2020)

Comparison with non-Hermitian SSH model

- Non-Hermitian SSH model

$$H_k = (t_1 + t_2 \cos k)\sigma_x + (t_2 \sin k + i\frac{\gamma}{2})\sigma_y$$

- Quantum walk with skin effect

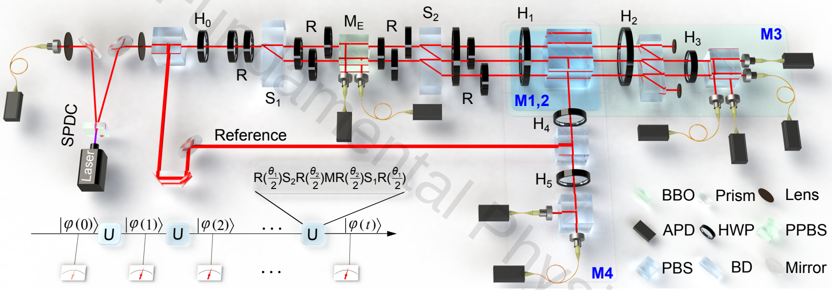
$$\begin{aligned} U_k &= d_0\sigma_0 - id_1\sigma_x - id_2\sigma_y - id_3\sigma_z \\ d_0 &= -\cosh\gamma \sin\theta_1 \sin\theta_2 + \cosh\gamma \cos k \cos\theta_1 \cos\theta_2 + i \sinh\gamma \cos\theta_1 \sin k \\ d_1 &= 0, \\ d_2 &= \cosh\gamma \cos\theta_1 \sin\theta_2 + \cos k \cosh\gamma \cos\theta_2 \sin\theta_1 + i \sin k \sinh\gamma \sin\theta_1 \\ d_3 &= -\sin k \cosh\gamma \cos\theta_2 + i \cos k \sinh\gamma \end{aligned}$$

In the low-k limit

$$\begin{aligned} d_2 &= a_2 + b_2 \cos k \\ d_3 &= a_3 \sin k + ib_3 \end{aligned}$$

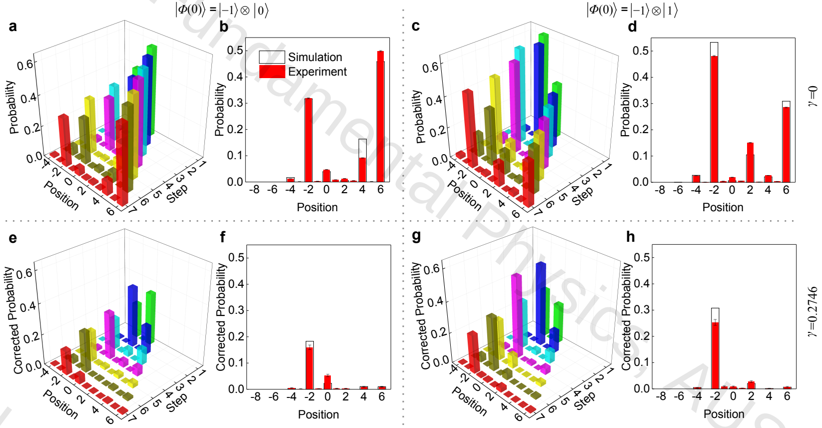
- Lattice model still quite different

Experimental implementation and observation



- Single photons

Experimental implementation and observation



- Single photons
- Non-Hermitian skin effect:
  probability localization in the absence of topological edge states

How to detect edge states?

- Time-dependent wave function

  $|\Phi(t)\rangle = U^t|\Phi(0)\rangle = \sum_n e^{-iE_n t}\Phi_n|\psi_n\rangle$

  with $(\Phi_n = \langle\chi_n|\Phi(0)\rangle$

- Time-integrated wave function

  $|\Phi_\epsilon(t)\rangle = \sum_{t'=0}^{t} \frac{e^{i\epsilon t'}}{t+1}|\Phi(t')\rangle \quad (\epsilon = 0, \pi)$

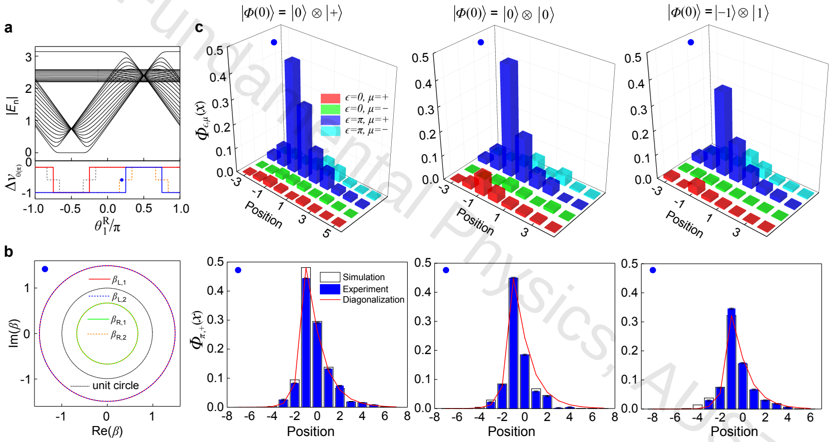
- Only components with $\epsilon = 0, \pi$ remain in the summation

  $|\Phi_\epsilon(t)\rangle = \sum_n f_\epsilon(E_n)\Phi_n|\psi_n\rangle$

- Full resolution of the edge states

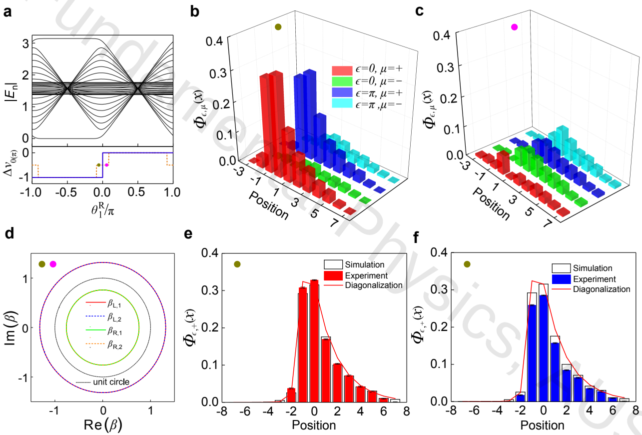
  $\Phi_{\epsilon,\mu}(x) = \left|\Big(\langle x| \otimes \langle\mu|\Big)|\Phi_\epsilon(t)\rangle\right| \quad (\mu = \pm)$

Confirming non-Hermitian bbc



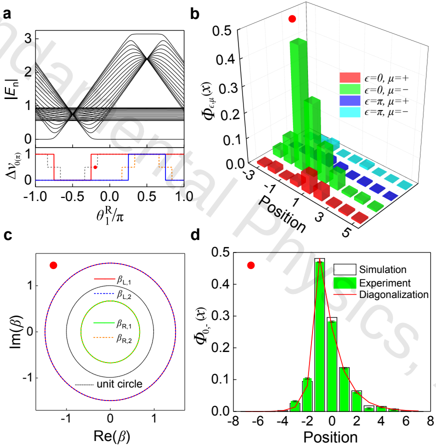
Edge states with $\epsilon = \pi$

Confirming non-Hermitian bbc



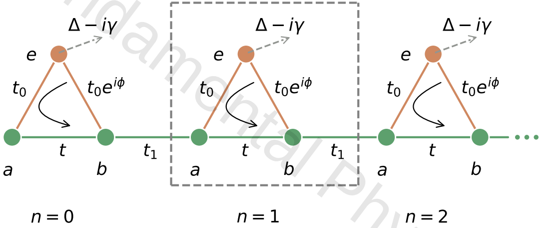
Edge states with $\epsilon = 0, \pi$

Confirming non-Hermitian bbc



Edge states with $\epsilon = 0$

In Cold Atoms: Aharonov-Bohm chain

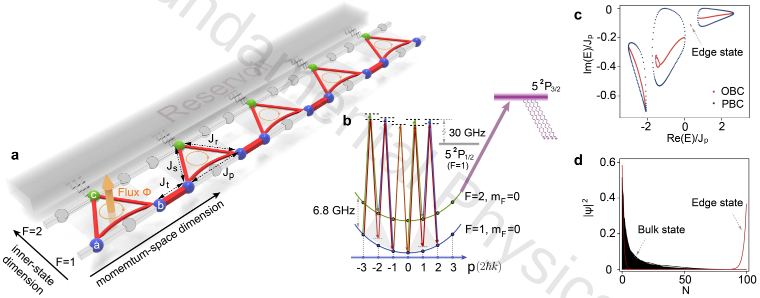


- Decoupled limit $(\Delta, \gamma \gg t, t_0, t_1)$

$$H_{\text{skin}} = \sum_m \left[\tilde{\Delta}(c^\dagger_{m,a}c_{m,a} + c^\dagger_{m,b}c_{m,b}) + t_1(c^\dagger_{m+1,a}c_{m,b} + \text{H.c.}) + (t+\tilde{\gamma})c^\dagger_{m,a}c_{m,b} + (t-\tilde{\gamma})c^\dagger_{m,b}c_{m,a}\right]$$

W. Gou et al., Phys. Rev. Lett. 124, 070402 (2020)

In Cold Atoms: Aharonov-Bohm chain

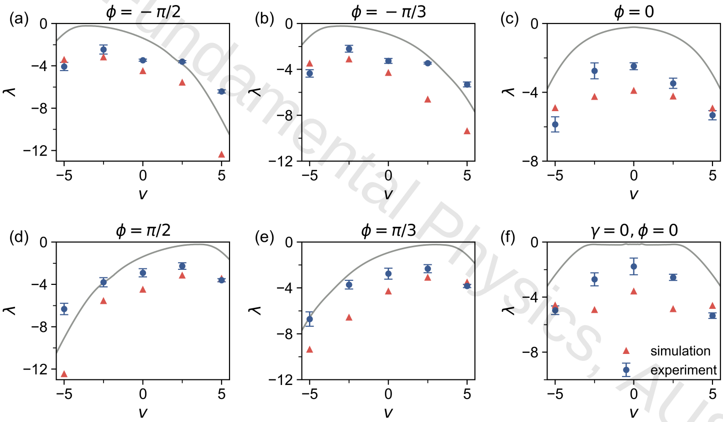


- AB chain in synthetic dimensions
- Laser-induced loss

Q. Liang et al., Phys. Rev. Lett. 129, 070401 (2022)

H. Li, WY, Phys. Rev. A 106, 053311 (2022)

In Cold Atoms: Aharonov-Bohm chain

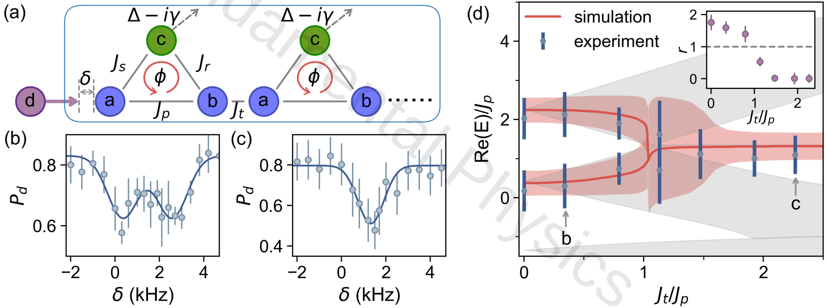


- Growth-rate and spectral measurements

Q. Liang et al., Phys. Rev. Lett. 129, 070401 (2022)

H. Li, WY, Phys. Rev. A 106, 053311 (2022)

In Cold Atoms: Aharonov-Bohm chain



- Growth-rate and spectral measurements

Q. Liang et al., Phys. Rev. Lett. 129, 070401 (2022)

H. Li, WY, Phys. Rev. A 106, 053311 (2022)

Summary and outlook

Revealing non-Hermitian physics through dynamics

- Topological and critical dynamics near exceptional points
- Non-Hermitian-skin-effect-related phenomena
- Many-body scenario?